Deret Fourier

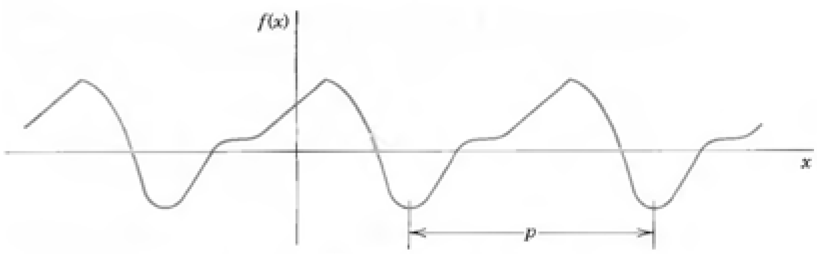
Fourier series are the basic tool for representing periodic functions. Deret Fourier merupakan penguraian fungsi periodik menjadi jumlahan fungsi-fungsi berosilasi, yaitu fungsi sinus dan kosinus, ataupun eksponensial kompleks.

Fungsi periodik :

$f(x+p) = f(x)$ untuk semua nilai x

p adalah periode

$f(x+np) = f(x)$ n = 1, 2, 3, ...



Periodic function

Deret Fourier :

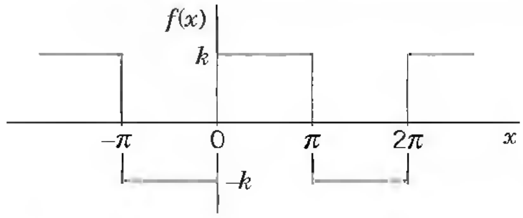
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right)$$

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\, dx$$

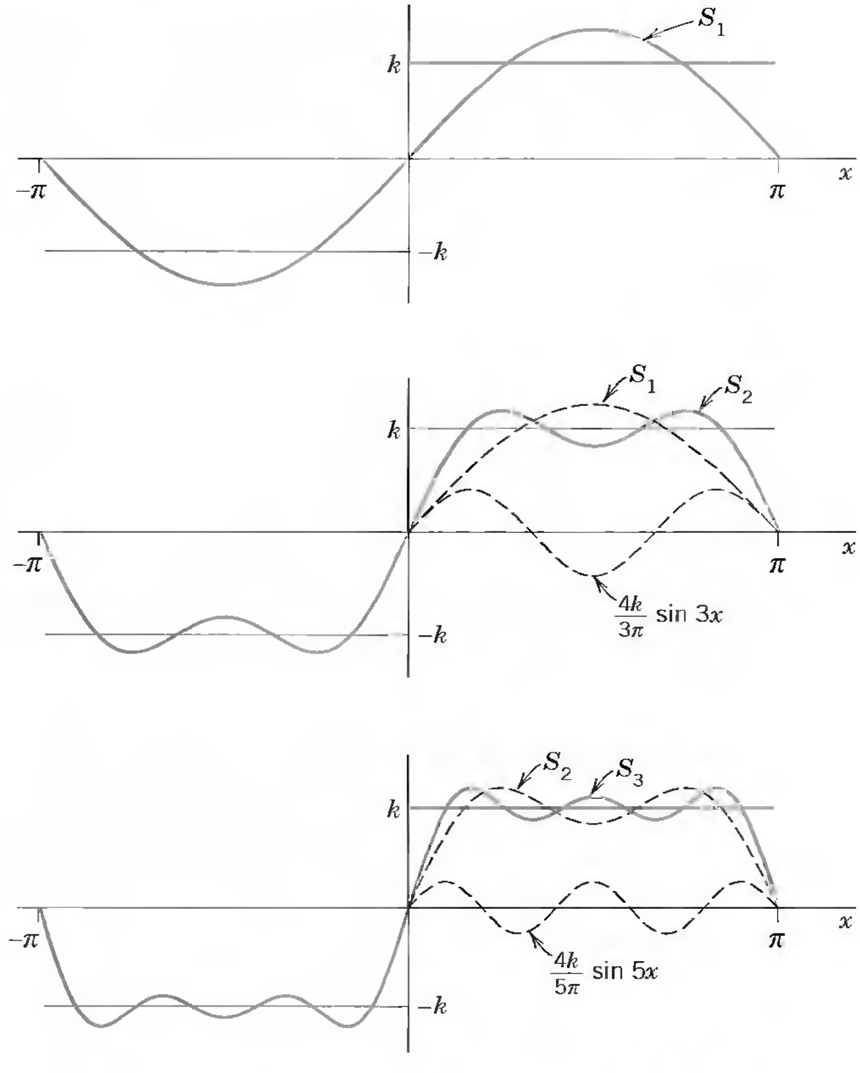
$$\left.\begin{aligned} a_n &= \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx \\ b_n &= \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx \end{aligned}\right\} \quad n = 1,2,3,\ldots$$

$a_0, a_1, a_2 b_1, b_2, \ldots$ adalah koefisien deret

Contoh:



(a) The given function $f(x)$ (Periodic rectangular wave)



(b) The first three partial sums of the corresponding Fourier series

Fungsi Genap dan Fungsi Ganjil

Fungsi Genap: $f(-x) = f(x)$ **Fungsi Ganjil:** $f(-x) = -f(x)$

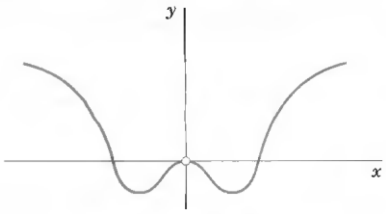


Fig. 262. Even function

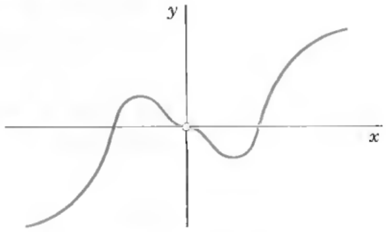


Fig. 263. Odd function

Fourier Cosine Series, Fourier Sine Series

The Fourier series of an **even** *function of period* $2L$ *is a* **"Fourier cosine series"**

(1) $$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{L} x \qquad (f \text{ even})$$

with coefficients (note: integration from 0 to L only!)

(2) $$a_0 = \frac{1}{L} \int_0^L f(x)\,dx, \qquad a_n = \frac{2}{L} \int_0^L f(x) \cos \frac{n\pi x}{L}\,dx, \qquad n = 1, 2, \cdots.$$

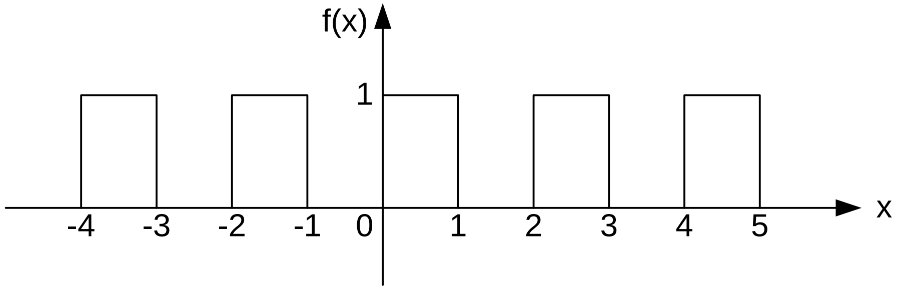
The Fourier series of an **odd** *function of period* $2L$ *is a* **"Fourier sine series"**

(3) $$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{L} x \qquad (f \text{ odd})$$

with coefficients

(4) $$b_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L}\,dx.$$

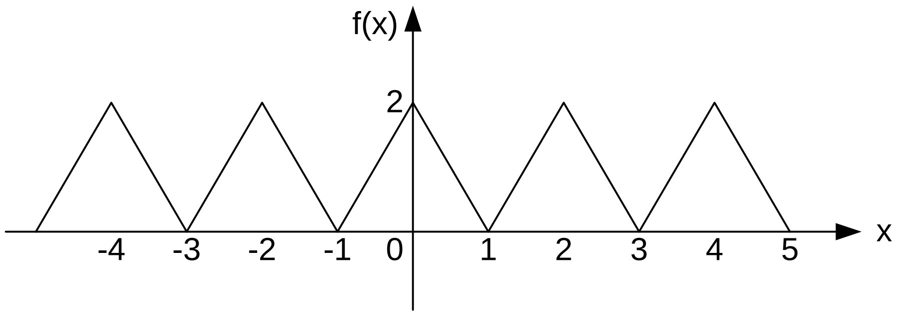
Latihan :



$$f(x)=\begin{cases}0 & ;-1<x<0\\ 1 & ;\ \ 0<x<1\end{cases}$$

atau

$$f(x)=\begin{cases}1 & ;0<x<1\\ 0 & ;1<x<2\end{cases}$$

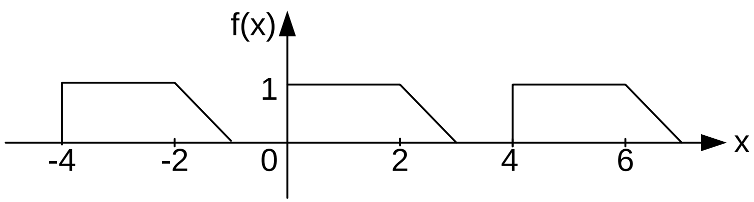


$$f(x)=\begin{cases}2x+2 & ;-1<x<0\\ -2x+2 & ;\ \ 0<x<1\end{cases}$$

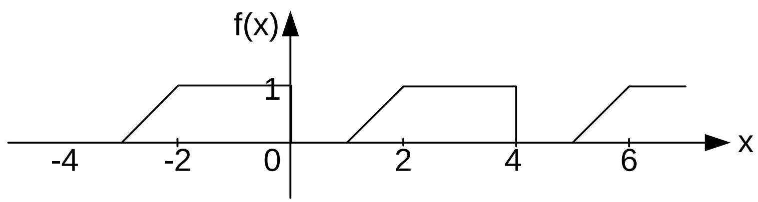
atau

$$f(x)=\begin{cases}-2x+2 & ;0<x<1\\ 2x-2 & ;1<x<2\end{cases}$$

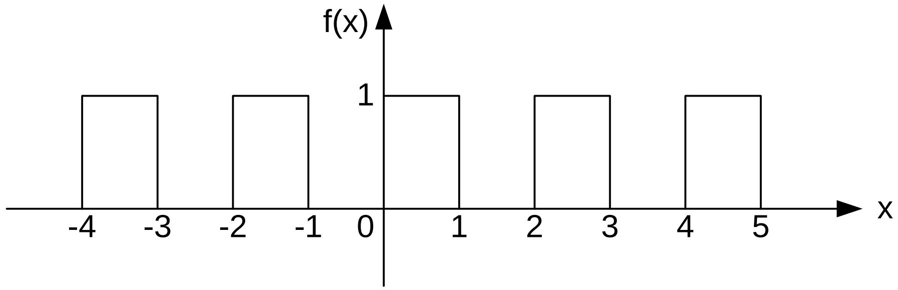
Tugas :



NIM GANJIL



NIM GENAP



$$f(x)=\begin{cases}0 & ;-1<x<0\\ 1 & ;\ \ 0<x<1\end{cases}$$

atau

$$f(x)=\begin{cases}1 & ;0<x<1\\ 0 & ;1<x<2\end{cases}$$

$$a_0=\frac{1}{L}\int_{-L}^{L} f(x)dx$$

$$a_0=\frac{1}{1}\left\{\int_{-1}^{0}0.dx+\int_{0}^{1}1.dx\right\}$$

$$a_0=\int_{0}^{1}1.dx$$

$$a_0=x\Big|_0^1=1$$

$$a_n=\frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$a_n=\frac{1}{1}\left\{\int_{-1}^{0}0.\cos\left(\frac{n\pi x}{1}\right)dx+\int_{0}^{1}1.\cos\left(\frac{n\pi x}{1}\right)dx\right\}$$

$$a_n=\int_{0}^{1}\cos(n\pi x)dx$$

$$a_n=\frac{\sin(n\pi x)}{n\pi}\Bigg|_0^1=\frac{\sin(n\pi)}{n\pi}=0$$

$$b_n=\frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$b_n=\frac{1}{1}\left\{\int_{-1}^{0}0.\sin\left(\frac{n\pi x}{1}\right)dx+\int_{0}^{1}1.\sin\left(\frac{n\pi x}{1}\right)dx\right\}$$

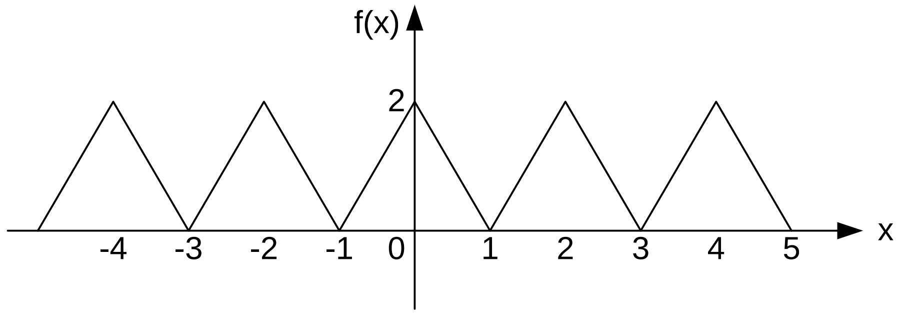
$$b_n=\int_{0}^{1}\sin(n\pi x)dx$$

$$b_n=-\frac{\cos(n\pi x)}{n\pi}\Bigg|_0^1=-\frac{\cos(n\pi)-1}{n\pi}=\frac{1-\cos(n\pi)}{n\pi}$$

$$\left.\begin{aligned}a_0&=1\\ a_n&=\frac{\sin(n\pi)}{n\pi}\\ b_n&=\frac{1-\cos(n\pi)}{n\pi}\end{aligned}\right\}\quad n=1,2,3,\ldots$$

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cos\left(\frac{n\pi x}{L}\right)+b_n\sin\left(\frac{n\pi x}{L}\right)\right)$$

$$f(x)=\frac{1}{2}+\sum_{n=1}^{\infty}\left\{\left(\frac{\sin(n\pi)}{n\pi}\cos(n\pi x)\right)+\left(\frac{1-\cos(n\pi)}{n\pi}\sin(n\pi x)\right)\right\}$$



$$f\left(x\right)=\begin{cases}2x+2 & ;-1<x<0\\ -2x+2 & ;\ 0<x<1\end{cases}$$

atau

$$f\left(x\right)=\begin{cases}-2x+2 & ;0<x<1\\ 2x-2 & ;1<x<2\end{cases}$$

$$a_0=\frac{1}{L}\int_{-L}^{L} f\left(x\right)dx$$

$$a_0=\frac{1}{1}\left\{\int_{-1}^{0}\left(2x+2\right)dx+\int_{0}^{1}\left(-2x+2\right)dx\right\}$$

$$a_0=\int_{-1}^{0}\left(2x+2\right)dx+\int_{0}^{1}\left(-2x+2\right)dx$$

$$a_0=\left(x^2+2x\Big|_{-1}^{0}\right)+\left(-x^2+2x\Big|_{0}^{1}\right)$$

$$a_0=\left(0-\left(1-2\right)\right)+\left(-1+2-0\right)$$

$$a_0=2$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$a_n = \frac{1}{1}\left\{\int_{-1}^{0}(2x+2)\cos\left(\frac{n\pi x}{1}\right)dx + \int_{0}^{1}(-2x+2)\cos\left(\frac{n\pi x}{1}\right)dx\right\}$$

$$a_n = \int_{-1}^{0}(2x+2)\cos(n\pi x)\,dx + \int_{0}^{1}(-2x+2)\cos(n\pi x)\,dx$$

$$\int_{-1}^{0}(2x+2)\cos(n\pi x)\,dx = \left.\frac{(2x+2)\sin(n\pi x)}{n\pi}\right|_{-1}^{0} - \int_{-1}^{0}\frac{2\sin(n\pi x)}{n\pi}dx$$

$$\int_{-1}^{0}(2x+2)\cos(n\pi x)\,dx = \left.\frac{(2x+2)\sin(n\pi x)}{n\pi}\right|_{-1}^{0} + \left.\frac{2\cos(n\pi x)}{(n\pi)^2}\right|_{-1}^{0}$$

$$\int_{-1}^{0}(2x+2)\cos(n\pi x)\,dx = 0 + \frac{2-2\cos(-n\pi)}{(n\pi)^2} = \frac{2-2\cos(n\pi)}{(n\pi)^2}$$

$$\int_{0}^{1}(-2x+2)\cos(n\pi x)\,dx = \left.\frac{(-2x+2)\sin(n\pi x)}{n\pi}\right|_{0}^{1} + \int_{0}^{1}\frac{2\sin(n\pi x)}{n\pi}dx$$

$$\int_{0}^{1}(-2x+2)\cos(n\pi x)\,dx = \left.\frac{(-2x+2)\sin(n\pi x)}{n\pi}\right|_{0}^{1} - \left.\frac{2\cos(n\pi x)}{(n\pi)^2}\right|_{0}^{1}$$

$$\int_{0}^{1}(-2x+2)\cos(n\pi x)\,dx = 0 - \frac{2\cos(n\pi)-2}{(n\pi)^2} = \frac{2-2\cos(n\pi)}{(n\pi)^2}$$

$$a_n = \int_{-1}^{0}(2x+2)\cos(n\pi x)\,dx + \int_{0}^{1}(-2x+2)\cos(n\pi x)\,dx$$

$$a_n = \frac{2-2\cos(n\pi)}{(n\pi)^2} + \frac{2-2\cos(n\pi)}{(n\pi)^2} = 4\left(\frac{1-\cos(n\pi)}{(n\pi)^2}\right)$$

$$a_n = \begin{cases} 0, \text{ n=genap} \\ \dfrac{8}{(n\pi)^2}, \text{ n=ganjil} \end{cases}$$

$$b_n = \frac{1}{1}\left\{\int_{-1}^{0}(2x+2)\sin\left(\frac{n\pi x}{1}\right)dx + \int_{0}^{1}(-2x+2)\sin\left(\frac{n\pi x}{1}\right)dx\right\}$$

$$b_n = \int_{-1}^{0}(2x+2)\sin(n\pi x)\,dx + \int_{0}^{1}(-2x+2)\sin(n\pi x)\,dx$$

$$\int_{-1}^{0}(2x+2)\sin(n\pi x)\,dx = -\left.\frac{(2x+2)\cos(n\pi x)}{n\pi}\right|_{-1}^{0} + \int_{-1}^{0}\frac{2\cos(n\pi x)}{n\pi}dx$$

$$\int_{-1}^{0}(2x+2)\sin(n\pi x)\,dx = -\left.\frac{(2x+2)\cos(n\pi x)}{n\pi}\right|_{-1}^{0} + \left.\frac{2\sin(n\pi x)}{(n\pi)^2}\right|_{-1}^{0}$$

$$\int_{-1}^{0}(2x+2)\sin(n\pi x)\,dx = -\frac{2}{n\pi} + \frac{0-2\sin(-n\pi)}{(n\pi)^2} = -\frac{2}{n\pi} + \frac{2\sin(n\pi)}{(n\pi)^2} = \frac{2\sin(n\pi)-2n\pi}{(n\pi)^2}$$

$$\int_{0}^{1}(-2x+2)\sin(n\pi x)\,dx = -\left.\frac{(-2x+2)\cos(n\pi x)}{n\pi}\right|_{0}^{1} - \int_{0}^{1}\frac{2\cos(n\pi x)}{n\pi}dx$$

$$\int_{0}^{1}(-2x+2)\sin(n\pi x)\,dx = -\left.\frac{(-2x+2)\cos(n\pi x)}{n\pi}\right|_{0}^{1} - \left.\frac{2\sin(n\pi x)}{(n\pi)^2}\right|_{0}^{1}$$

$$\int_{0}^{1}(-2x+2)\sin(n\pi x)\,dx = -\frac{0-2}{n\pi} - \frac{2\sin(n\pi)}{(n\pi)^2} = \frac{2}{n\pi} - \frac{2\sin(n\pi)}{(n\pi)^2} = \frac{2n\pi-2\sin(n\pi)}{(n\pi)^2}$$

$$b_n = \int_{-1}^{0}(2x+2)\sin(n\pi x)\,dx + \int_{0}^{1}(-2x+2)\sin(n\pi x)\,dx$$

$$b_n = \frac{2\sin(n\pi)-2n\pi}{(n\pi)^2} + \frac{2n\pi-2\sin(n\pi)}{(n\pi)^2} = 0$$

$$\left.\begin{aligned} a_0 &= 2 \\ a_n &= 4\left(\frac{1-\cos(n\pi)}{(n\pi)^2}\right) \\ b_n &= 0 \end{aligned}\right\} \quad n = 1, 2, 3, \ldots$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos\left(\frac{n\pi x}{L}\right) + b_n\sin\left(\frac{n\pi x}{L}\right)\right)$$

$$f(x) = 1 + \sum_{n=1}^{\infty}4\left(\frac{1-\cos(n\pi)}{(n\pi)^2}\right)\cos(n\pi x)$$

Listing Program MATLAB

```
% % kotak
clear all
nn=10;
x=-3:.01:3;
L=1;
bn=0;
a0=1;
a0=a0/2*ones(1,length(x));
F=a0;
for n=1:nn
    an=sin(n*pi)/n/pi;
    bn=(1-cos(n*pi))/n/pi;
    A=an*cos(pi*n*x/L);
    B=bn*sin(pi*n*x/L);
    F=F+A+B;
end
figure;plot(x,F)
```

```
% % segitiga
clear all
nn=10;
x=-3:.01:3;
L=1;
bn=0;
a0=2;
a0=a0/2*ones(1,length(x));
F=a0;
for n=1:nn
    an=4*(1-cos(pi*n))/((n*pi)^2);
    bn=0;
    A=an*cos(pi*n*x/L);
    B=bn*sin(pi*n*x/L);
    F=F+A+B;
end
figure;plot(x,F)
```