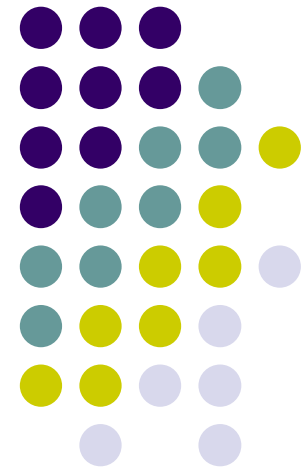
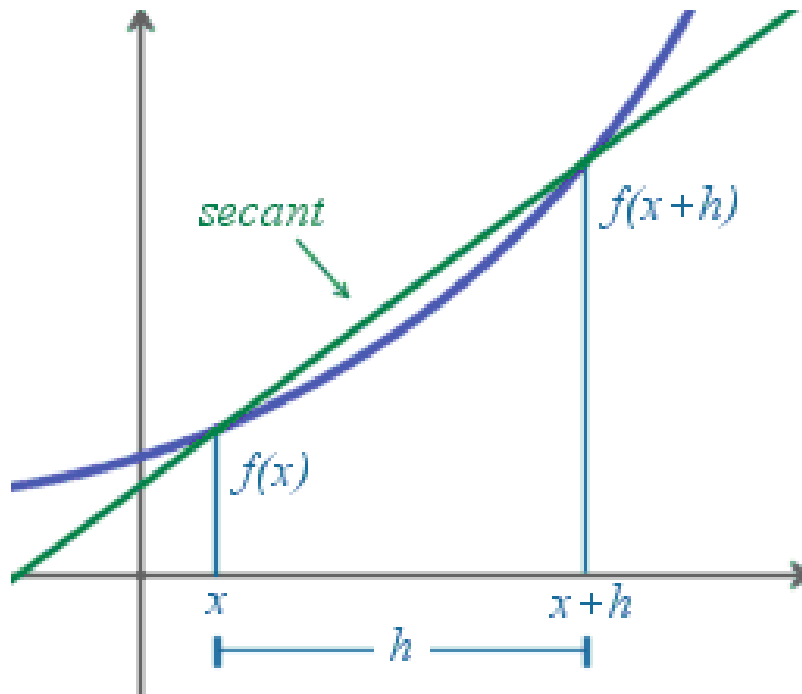
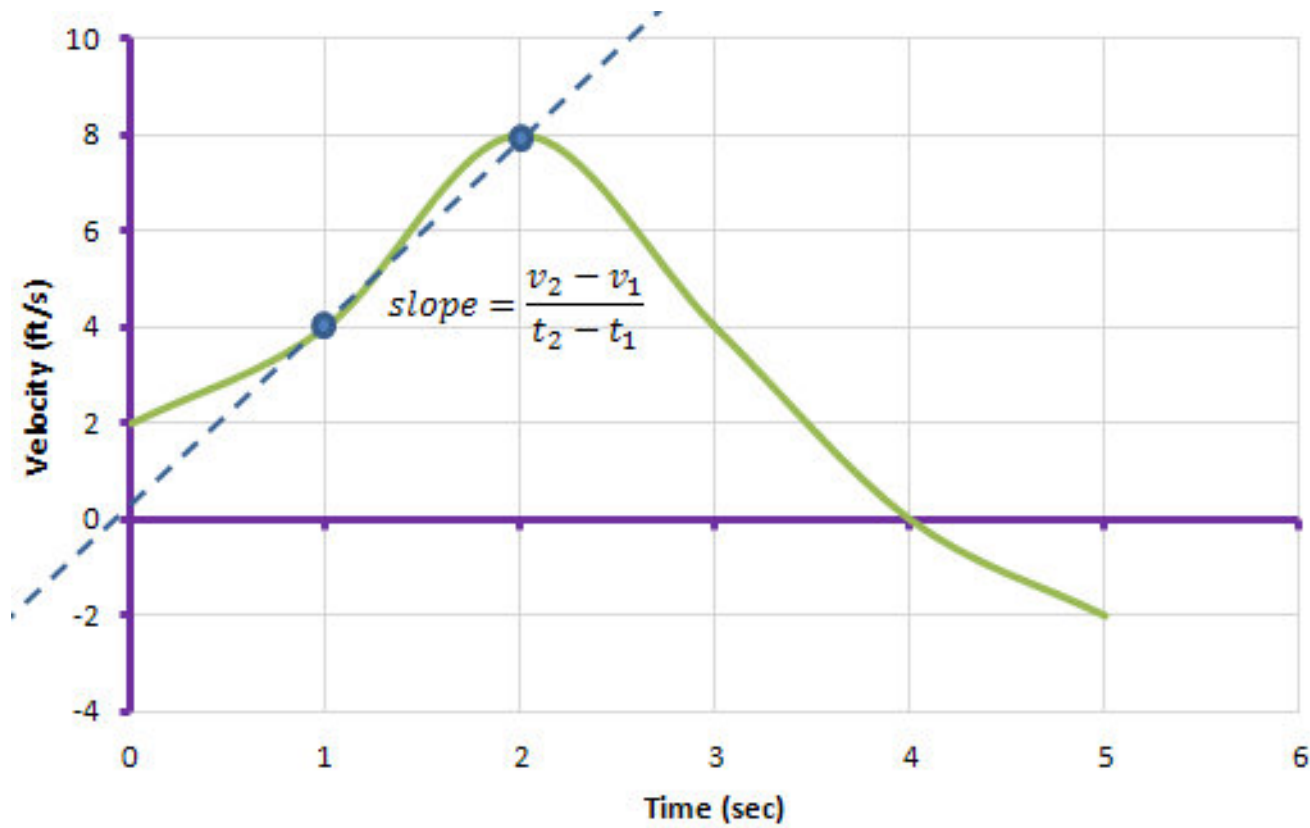


METODE NUMERIK

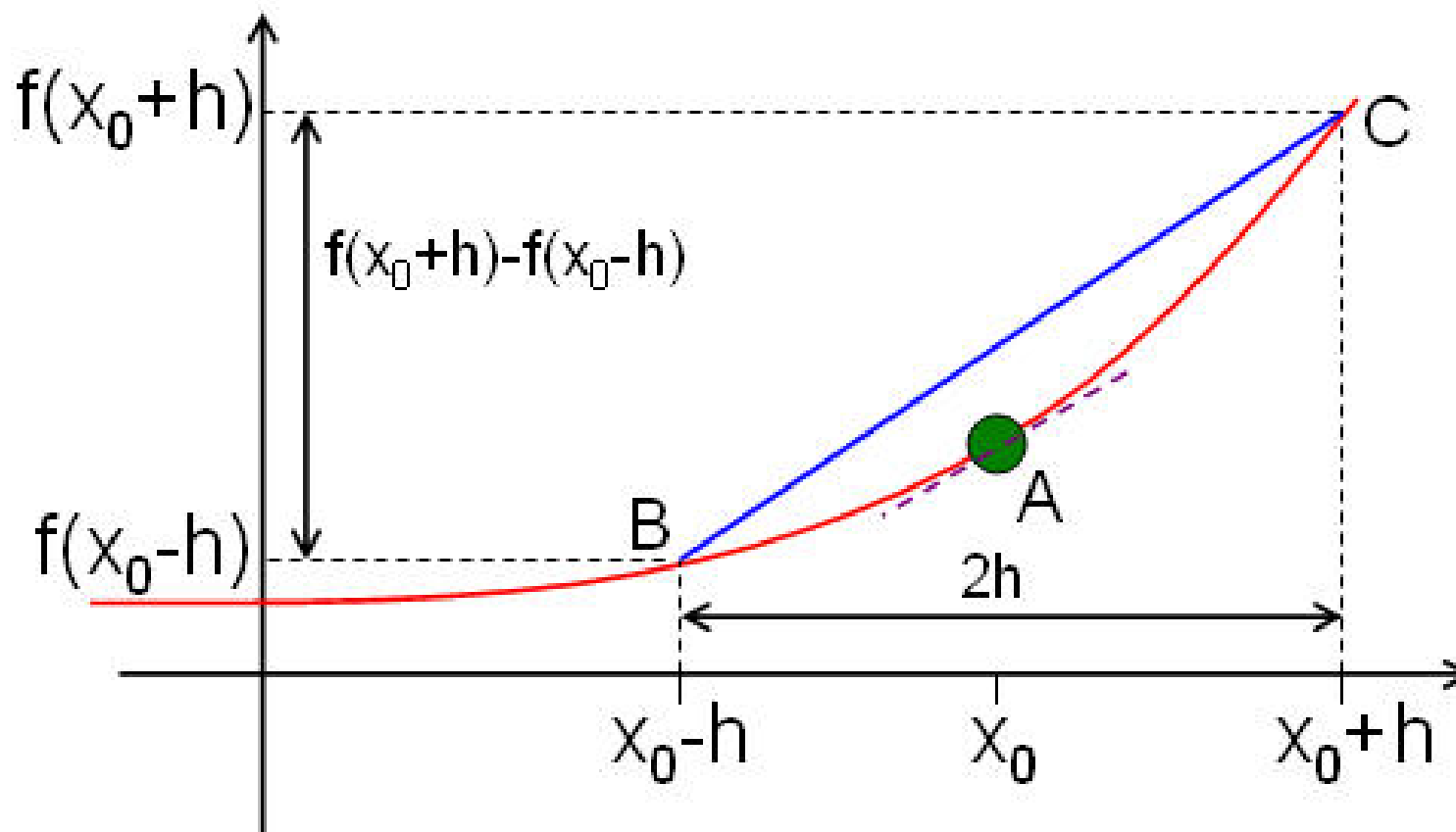
DIFERENSIASI NUMERIK



Apersepsi



Pendekatan Konsep Diferensiasi





Jenis-Jenis Metode Diferensiasi

- Metode Euler
- Metode Runge-Kutta
- Metode Taylor
- Metode Adam
- Metode Milne
- Metode Adam-Moulton
- Metode modifikasi Euler



Metode Euler

- Menghitung persamaan differensial melalui taksiran langsung dari slope x_i diberi turunan pertama.

$$y_{i+1} = y_i + f(x_i, y_i)h$$

Metode Euler (Ex.)



- Selesaikan persamaan differensial $\frac{dy}{dx} = x\sqrt{y}$ pada interval $x = 0$ s/d $x = 1$, $h = \frac{1}{4}$. Pada saat $x = 0$, nilai $y = 1$. Hitung kesalahan sebenarnya!



Metode Euler (Ex.)

- Untuk $x = 0 \rightarrow y = 1$

- Untuk $x = 0,25$

$$\begin{aligned}y_{i+1} &= y_i + f(x_i, y_i) \cdot h \\&= 1 + f(0,1) \cdot 0,25 \\&= 1 + 0\sqrt{1} \cdot 0,25 \\&= 1\end{aligned}$$

- Untuk $x = 0,5$

$$\begin{aligned}y_{i+1} &= y_i + f(x_i, y_i) \cdot h \\&= 1 + f(0,25;1) \cdot 0,25 \\&= 1 + 0,25\sqrt{1} \cdot 0,25 \\&= 1,0625\end{aligned}$$



Metode Euler (Ex.)

- Untuk $x = 0,75$

$$\begin{aligned}y_{i+1} &= y_i + f(x_i, y_i) \cdot h \\&= 1,0625 + f(0,5; 1,0625) \cdot 0,25 \\&= 1,0625 + 0,5\sqrt{1,0625} \cdot 0,25 \\&= 1,1914\end{aligned}$$

- Untuk $x = 1$

$$\begin{aligned}y_{i+1} &= y_i + f(x_i, y_i) \cdot h \\&= 1,1914 + f(0,75; 1,1914) \cdot 0,25 \\&= 1,1914 + 0,75\sqrt{1,1914} \cdot 0,25 \\&= 1,3961\end{aligned}$$

Metode Euler (Ex.)

x	y
0	1
0,25	1
0,5	1,0625
0,75	1,1914
1	1,3961





Metode Euler (Ex.)

- Nilai eksak

$$\frac{dy}{dx} = x\sqrt{y} \rightarrow dy = x\sqrt{y} \cdot dx$$

$$\frac{dy}{\sqrt{y}} - x \cdot dx = 0$$

$$y^{-1/2} \cdot dy - x \cdot dx = 0$$

$$\int y^{-1/2} \cdot dy - \int x \cdot dx = \int 0$$

$$2\sqrt{y} - \frac{1}{2}x^2 = C$$



Metode Euler (Ex.)

- Pada saat $x = 0$; $y = 1$

$$2\sqrt{1} - \frac{1}{2}(0)^2 = C \Rightarrow C = 2$$

- Persamaan

$$2\sqrt{y} - \frac{1}{2}x^2 = 2$$



Metode Euler (Ex.)

- Untuk $x = 0,25$

$$2\sqrt{y} - \frac{1}{2}(0,25)^2 = 2$$

$$2\sqrt{y} = 2,0325$$

$$\sqrt{y} = 1,015625$$

$$y = 1,0315$$



Metode Euler (Ex.)

- Untuk $x = 0,5$

$$2\sqrt{y} - \frac{1}{2}(0,5)^2 = 2$$

$$\sqrt{y} = 1,0625$$

$$y = 1,1289$$



Metode Euler (Ex.)

- Untuk $x = 0,75$

$$2\sqrt{y} - \frac{1}{2}(0,75)^2 = 2$$

$$\sqrt{y} = 1,140625$$

$$y = 1,3010$$



Metode Euler (Ex.)

- Untuk $x = 1$

$$2\sqrt{y} - \frac{1}{2}(1)^2 = 2$$

$$\sqrt{y} = 1,25$$

$$y = 1,5625$$



Metode Euler (Ex.)

x	y_{Euler}	y_{sebenarnya}	ε_t
0	1	1	0 %
0,25	1	1,0315	3,0538 %
0,5	1,0625	1,1289	5,8818 %
0,75	1,1914	1,3010	8,4243 %
1	1,3961	1,5625	10,6496 %

$$\varepsilon_t = \left| \frac{y_{Euler} - y_{sebenarnya}}{y_{sebenarnya}} \right| * 100\%$$



Metode Heun

- Untuk memperbaiki Metode Euler, digunakan Metode Heun dengan cara perbaikan dari perkiraan nilai slopenya.
- Perbaikan perkiraan slope tersebut, ditempuh melalui nilai turunan dari slope-nya pada titik awal. Kemudian mencari turunan slope-nya pada titik akhir dan nilai tersebut dirata-ratakan.



Metode Heun

- Langkah-langkah Metode Heun:
 1. Mencari slope awal = $f(x_i, y_i)$
 2. Slope awal pada no.1 digunakan untuk ekstrapolasi nilai y_{i+1}^o , dengan rumus

$$y_{i+1}^o = y_i + f(x_i, y_i) \cdot h$$



Metode Heun

3. Persamaan prediktor (y_{i+1}^o) digunakan untuk mencari slope akhir (sebut dengan y_{i+1}^l), dengan rumus

$$y_{i+1}^l = f\left(x_{i+1}; y_{i+1}^o\right)$$

4. Mencari slope rata-rata (sebut dengan $\bar{y}'$)

$$\bar{y}' = \frac{f(x_i, y_i) + f\left(x_{i+1}, y_{i+1}^o\right)}{2}$$



Metode Heun

5. Slope rata-rata ini yang sebenarnya digunakan untuk mengekstrapolasikan y_i ke y_{i+1}

$$y_{i+1} = y_i + (\text{slope rata-rata}) \cdot h$$

$$y_{i+1} = y_i + \frac{f(x_i, y_i) + f\left(x_{i+1}, y_{i+1}^o\right)}{2} \cdot h$$

Metode Heun (Ex.)



- Selesaikan persamaan differensial $\frac{dy}{dx} = x\sqrt{y}$ pada interval $x = 0$ s/d $x = 1$, $h = \frac{1}{4}$. Pada saat $x = 0$, nilai $y = 1$. Hitung kesalahan sebenarnya!



Metode Heun (Ex.)

Untuk $x = 0,25$

1. Slope awal = $f(x_i, y_i) = f(0,1) = 0\sqrt{1} = 0$
2. Mencari prediktor

$$\begin{aligned} y_{i+1}^{\circ} &= y_i + f(x_i, y_i) \cdot h \\ &= 1 + f(0,1) \cdot 0,25 \\ &= 1 + 0 \cdot 0,25 \\ &= \underline{1} \end{aligned}$$

3. Mencari slope akhir

$$f\left(x_{i+1}, y_{i+1}^{\circ}\right) = f(0,25,1) = 0,25\sqrt{1} = \underline{\underline{0,25}}$$

4. Mencari slope rata-rata

$$\bar{y}' = \frac{f(x_i, y_i) + f\left(x_{i+1}, y_{i+1}^{\circ}\right)}{2} = \frac{0 + 0,25}{2} = \underline{\underline{0,125}}$$

5. $y_{i+1} = y_i + \bar{y}' \cdot h = 1 + 0,125 \cdot 0,25 = \underline{\underline{1,0313}}$



Metode Heun (Ex.)

Untuk $x = 0,5$

1. Slope awal : $f(x_i, y_i) = f(0,25; 1,0313) = 0,25\sqrt{1,0313} = 0,25388$

2. Prediktor :

$$\begin{aligned} y_{i+1}^o &= y_i + f(x_i, y_i) \cdot h \\ &= 1,0313 + f(0,25; 1,0313) \cdot 0,25 \\ &= 1,0313 + 0,25\sqrt{1,0313} \cdot 0,25 \\ &= 1,0948 \end{aligned}$$

3. Slope akhir :

$$f(x_{i+1}, y_{i+1}^o) = f(0,5; 1,0948) = 0,5\sqrt{1,0948} = 0,52316$$

4. Slope rata-rata :

$$\bar{y}' = \frac{f(x_i, y_i) + f(x_{i+1}, y_{i+1}^o)}{2} = \frac{0,25388 + 0,52316}{2} = 0,38852$$

5. $y_{i+1} = y_i + \bar{y}' \cdot h = 1,0313 + 0,38852 \cdot 0,25 = 1,1284$



Metode Heun (Ex.)

Untuk $x = 0,75$

1. Slope awal : $f(x_i, y_i) = f(0,5; 1,1284) = 0,5\sqrt{1,1284} = 0,53113$

2. Prediktor :

$$\begin{aligned} y_{i+1}^o &= y_i + f(x_i, y_i) \cdot h \\ &= 1,1284 + f(0,5; 1,1284) \cdot 0,25 \\ &= 1,2612 \end{aligned}$$

3. Slope akhir :

$$f(x_{i+1}, y_{i+1}^o) = f(0,75; 1,2612) = 0,75\sqrt{1,2612} = 0,84227$$

4. Slope rata-rata :

$$\bar{y}' = \frac{f(x_i, y_i) + f(x_{i+1}, y_{i+1}^o)}{2} = \frac{0,53113 + 0,84227}{2} = 0,68670$$

5. $y_{i+1} = y_i + \bar{y}' \cdot h = 1,1284 + 0,68670 \cdot 0,25 = 1,3001$



Metode Heun (Ex.)

Untuk $x = 1$

1. Slope awal : $f(x_i, y_i) = f(0,75;1,3001) = 0,75\sqrt{1,3001} = 0,85516$

2. Prediktor :

$$\begin{aligned} y_{i+1}^{\circ} &= y_i + f(x_i, y_i) \cdot h \\ &= 1,3001 + f(0,75;1,3001) \cdot 0,25 \\ &= 1,3001 + 0,85516 \cdot 0,25 \\ &= 1,5139 \end{aligned}$$

3. Slope akhir :

$$f(x_{i+1}, y_{i+1}^{\circ}) = f(1;1,5139) = 1\sqrt{1,5139} = 1,2304$$

4. Slope rata-rata :

$$\bar{y}' = \frac{f(x_i, y_i) + f(x_{i+1}, y_{i+1}^{\circ})}{2} = \frac{0,85516 + 1,2304}{2} = 1,0428$$

5. $y_{i+1} = y_i + \bar{y}' \cdot h = 1,3001 + 1,0428 \cdot 0,25 = 1,5608$



Metode Heun (Ex.)

x	y_{Heun}	$y_{\text{sebenarnya}}$	ε_t
0	1	1	0 %
0,25	1,0313	1,0315	0,01939 %
0,5	1,1284	1,1289	0,0443 %
0,75	1,3001	1,3010	0,06918 %
1	1,5608	1,5625	0,1088 %